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Time Preference and its Effects on Intertemporal Optimization of Economic Resources, A Critical Viewpoint

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Abstract:

Time preference has a peculiar role in determining the level of economic activities. Time preference is the most important origin of interest rate. In this paper we study the founders and defenders' viewpoints about time preference and then we try to criticize them. It seems that discounting future utilities is resulting from irrationality and it is ethically indefensible too. From mathematical aspect, discounting is necessary because it convergences sum of social welfare function to some finite number and so the optimal time path for consumption is attained. But this isn't true for a growing population. It seems that there aren't any rational reasons for preferring current generation's welfare to future generations'.

If we have time preference, in intertemporal allocation, it reduces available resources for future consumption and so the level of saving will be decreased. We study a numerical example for affecting time path of an ore extraction of a nonrenewable resource. Results show that the higher the time preference, the higher the rate of extraction will be occurred. Because with a high rate for time preference, present value of cash flows will be discounted more strongly and it increases the incentive to mine the more quickly

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1. Introduction:

Time preference has a peculiar role in determining interest rates and so it determines the level of economic activities. So studying the mechanics of time preference from the view of its determinants is very important. In this study we introduce the viewpoints of defenders and opponents of time preference and then we try to criticize them. After defining the concept of time preference in the second section, we study its relation to interest rate via a mathematical model in the third section. In the forth section of paper we study time preference from the viewpoints of its founders as like Bohm-Bawerk, Irving Fisher and Von Mises. The effects of positive time preference will be also introduced numerically. We try to simplify this with spreadsheet facilities.

2. Time Preference:

Time preference (or discounting) pertains to how large a premium a consumer will place on enjoyment nearer in time over more remote enjoyment.

Someone with a high time preference is focused substantially on his well-being in the present and the immediate future relative to the average person, while someone with low time preference places more emphasis than average on their well-being in the further future.

Time preferences are captured mathematically in the discount function. The higher the time preference, the higher the discount placed on returns receivable or costs payable in the future.

The time preference that an individual exhibits at any given moment is determined solely by their personal preferences. As such, if one prefers to save his money but cannot do so in the present, he is still considered to have a low time preference. One of the factors that may determine an individual's time preference is how long that individual has lived. An older individual may have a lower time preference (relative to what he had earlier in life) due to a higher income and to the fact that he has had more time to acquire durable commodities (such as a college education or a house).

The rate of time preference is usually taken as a parameter in an individual's utility function which captures the trade off between consumption today and consumption in the future, and is thus exogenous and subjective. It is also the underlying determinant of the real rate of interest. The rate of return on investment is generally seen as return on capital, with the real rate of interest equal to the marginal product of capital at any point in time. Consumers, who are facing a choice between consumption and saving, respond to the difference between the market interest rate and their own subjective rate of time preference ("impatience") and increase or decrease their current consumption according to this difference.

3. Time Preference and Interest Rate:

The time preference theory of interest is an attempt to explain interest through the demand for accelerated satisfaction. This is particularly important in microeconomics.

In the neoclassical theory of interest due to Irving Fisher, the interest rate determines the relative price of present and future consumption. Time preference, in conjunction with relative levels of present and future consumption, determines the marginal rate of substitution between present and future consumption. These two rates must necessarily be equal, and this equilibrium is brought about by the relative prices of present and future consumption.

Following Olson & Baily (1981), we represent utility function of a representative consumer as:

$$V = U(C_0) + \rho . U(C_1) \tag{1}$$

Consumer lives for two periods (present and future). C_0 is his present consumption and C_1 is future consumption.

Now consider a consumer wants to choose among alternative combinations (C_0, C_1) of consumption at t=0 and t=1, holding its combination pattern for all later periods fixed. If we differentiate (1) we have:

$$dV = U'(C_0)dC_0 + \rho.U'(C_1)dC_1 \quad (2)$$

To remain constant utility, it is necessary to have $dV = 0$. So:

$$U'(C_0)dC_0 + \rho.U'(C_1)dC_1 = 0$$

Or:

$$\left|\frac{dC_1}{dC_0}\right| = \frac{U'(C_0)}{\rho.U'(C_1)} \quad (3)$$

Consumer will demand an increase in C_1 greater than a reduction in C_0, that is:

$$\left|\frac{dC_1}{dC_0}\right| > 1$$

For holding utility constant, if either of two things is true:

$$C_1 > C_0 \text{ with } \rho = 1 \quad (4)$$

Or

$$\rho < 1 \text{ with } C_1 = C_0 \quad (5)$$

These are two conditions that state the two causes of interest in Bohm-Bawerk's viewpoint.

Inequality (4) follows because of the $U'' < 0$. Diminishing Marginal Utility of income implies that, if all else is equal, a consumer with greater consumption in the next period will be willing to borrow at a positive interest rate in other to make consumption in the two periods more nearly equal. This is the first cause of the preference for present over future consumption mentioned in the quotation from Bohm-Bawerk.

The second cause is represented by the parameter ρ in the utility function. When it is less than one, consumer would be willing to pay a positive interest rate for increasing first period consumption at the expense of his future consumption.

4. The Defense of Time Preference:

Founders of time preference have some powerful reasons for protecting of time preference. From the viewpoint of Irving Fisher individuals have a kind of impatient nature. So they prefer present over future. Therefore interest is a premium for an individual who can conquest on this blood. Ludwig Von Mises believes that time preference is a part of human action and the most important reason for existing time preference is the nature of consuming. Consuming is a reason for existing time preference. Because if we haven't any time preference we didn't consume at all and we will delay consumption to tomorrow and so on. If we did not have time preference, we would never consume anything, because we would keep delaying consumption. As Mises puts it (Mises, 1949, P. 486):

"We must conceive that a man who does not prefer satisfaction within a nearer period of the future to that in a remoter period would never achieve consumption and enjoyment at all."

As Mises believes, satisfaction of a want in the nearer future is preferred to that in the farther distant future. Present goods are more valuable than future goods (Mises, 1949, P. 484).

Eugen von Boehm-Bawerk built upon the time-preference ideas of Carl Menger, insisting that there is always a difference in value between present goods and future goods of equal quality, quantity, and form. Furthermore, the value of future goods diminishes as the length of time necessary for their completion increases.

Boehm-Bawerk cited three reasons for this difference in value. First of all, in a growing economy, the supply of goods will always be larger in the future than it is in the present. Secondly, people have a tendency to underestimate their future needs due to carelessness and shortsightedness. Finally entrepreneurs would rather initiate production with goods presently available, instead of waiting for future goods and delaying production.

During past 50 years, economists who are defenders Bohm-Bawerk's opinion have studied more on the theory of time preference. They have tried to update the viewpoints of past founders of time preference. For example we can name from Barro (1974), Eckstein (1957), Marglin (1963), Cass (1965, 1966) and Koopmans (1960, 1964, 1972). They tried to justify the existence of time preference in intertemporal social welfare function via discount factor. In the other words, they tend to set out time preference rate ($(0 < \rho < 1)$ in the intertemporal social welfare and then justify it:

$$S = \int_0^\infty U(c)e^{-\rho t}dt \qquad (6)$$

Including the time discount rate into our social welfare function would certainly make things easier. But how are we to justify it? If we follow Pigou, Ramsey and company in their reasoning, there seems to be no "ethical" justification for putting a utility discount into the social welfare function. How might this be disputed?

The absence of time preference implies incompleteness of social preference orderings. Because it may be impossible to compare alternative consumption path and say which one is socially better. With an infinite time horizon it is quite possible that there are feasible consumption paths such that S=0. If to consumption paths each generates an S which is infinite, then they become incomparable.

So from mathematical aspect it is necessary to include some positive discounting factor to convergence social ordering to some finite number.

5. The attack to Time preference

5.1. Pigou's argument:

From this proposition, the Cambridge economist Arthur C. Pigou (1920) posed an interesting conundrum: if, indeed, agents tend to underestimate their future utility, they will probably not make proper provision for their future wants and thus personally save less than they would have wished had they made the calculation correctly. In other words, Pigou proposed, the very fact that people possess defective "telescopic faculties" probably means that savings, as a whole, are less than what is "optimal". This, Pigou conjectured, implies that there is a "market failure" of sorts in the market for savings.

"Generally speaking, everybody prefers present pleasures or satisfactions of given magnitude to future pleasures or satisfactions of equal magnitude, even when the latter are perfectly certain to occur. But this preference for present pleasures does not -- the idea is self-contradictory -- imply that a present pleasure of given magnitude is any greater than a future pleasure of the same magnitude. It implies only that our telescopic faculty is defective, and those we, therefore, see future pleasures, as it were; on a diminished scale. This reveals a far-reaching economic disharmony. For it implies that people distribute their resources between the present, the near future, and the remote future on the basis of a wholly irrational preference." (A.C. Pigou, Economics of Welfare, 1920: p.24-5).

5.2. Ramsey's argument:

Ramsey deliberately excluded discounting of future utility form this social welfare function: just because people are individually short-sighted, does not mean that society should be similarly "short-sighted". This is a normative, not a positive exercise.

As outlined in our introduction, Arthur C. Pigou's (1920) assertion that myopic agents might "save too little" was taken up by Frank P. P. Ramsey (1928). Ramsey's main concern was to determine the optimal rate of savings and then show how myopic agents would not achieve that optimum.

Frank Ramsey considered time discounting as "a practice which is ethically indefensible and arises merely from the weakness of the imagination" (Ramsey, 1928). Time preference, as Pigou (1920: Pt I, Ch. 2) originally asserted, is a personal weakness which should not be imported into a normative exercise.

However, Ramsey recognized that by omitting time preference, we can reach to the maximum level of saving that a society could be reach.

5.3. Incompleteness of Social Preference Orderings:

Cass-Koopmans reasoning for including time preference (or discounting) in intertemporal social welfare function is only a mathematical testimony with no economical reasoning. Including discount factor in social welfare function generates a non-ethical result. Because discounting will generate an inequality in intergenerational allocation process in favour of present generations. Does present generation have any permission to discount future generations' utility for getting rid off mathematical limitations? Of course Cass-Koopmans argument is a mathematical one and it is better to have a mathematical answer to it. This answer is provided by Weizsacker (1965) and Atsomi (1965).

Moreover, according to reasons for defense of discounting, everyone prefers his present to his future. This reason is not suitable for including discount factor in intertemporal social utility function. Because in a discounted social utility function we (as a social planner) prefer life of present generations (both their present and their future) to future generations. This action hasn't any relation to personal reasoning stated by founders of time preference. So there aren't any economical reasons for including time preference in social utility function. This is a normative exercise and not only it isn't defensible logically but also as Ramsey pointed out that is ethically indefensible!

5-4. Population Growth:

Nevertheless, if we don't accept all of assailant reasons and we want to discount future generations' utility, it can be used only for a static economy. In the other words, if population grows at a positive rate, the mathematical effect of discounting will be counterbalanced by the opposite effect of population growth.

For showing this effect, please let L(t) denote the number of people at time t. Let us suppose that population grows at the exponential rate n, so that population at time t can be expressed as:

$$L(t) = L_0 e^{nt} \tag{7}$$

where L_0 is the initial population. The growth rate of labor can also be written as $g_L = (\frac{dL}{dt})/L = n$. We add this, into our social welfare function as following:

$$S = \int_0^\infty e^{nt}.U(c)e^{-\rho t}dt$$

or:

$$S = \int_0^\infty e^{nt}.U(c)e^{(n-\rho)t}dt \tag{8}$$

Thus social welfare is discounted by the time preference rate ρ adjusted by the rate of population growth n. Thus, we can think of $(\rho - n)$ as the net discount rate.

The logic for this is: if we did not incorporate population growth into our discount factor then we would be punishing a single individual in the future twice -- once because he is in the future (and his forefathers were "myopic"), and twice because he belongs to a generation which is larger in number. In order to keep some sense of equal treatment across individuals in this social welfare function, we must adjust the discount rate for the population growth rate.

The use of $(\rho - n)$ as the discount rate is defended convincingly by Kenneth J. Arrow and Mordecai Kurz (1970: p.11-14).

However, for $S < \infty$, we need it that $(\rho > n)$, i.e. the rate of time preference must exceed the rate of population growth for the integral to converge. This is a necessary assumption, but not necessarily a very reasonable or intuitive one. If, as it turns out in the solution, ρ is equated with the rate of interest, then this convergence condition says that we need the rate of interest to exceed the natural rate of growth. But there aren't any economical reasons for this condition. If ρ is equated with the rate of interest, economists believe that in steady-state the rate of time preference is equal to the rate of population growth (Samuelson, 1958).

5.5. Some Other Criticizes:

A. Unlike Bohm-Bawerk's opinion about preferring present over future, we can find some alternatives which future is preferred to present. For example, an individual who hasn't enough profession for working with a technical tool, with an educational period, he prefers using it in future instead of present.

B. One of the reasons for preferring present over future is the uncertainty of future. So they believe that certain present is preferred to uncertain future. This reasoning is not a valid one. It maybe is an interesting sentence, but that is not a determinant for interest rate. Uncertainty hasn't any effect on interest rate. Because when a lender lends money to a borrower, however there exists an unpaid risk for lender and any interest rates can not omit this risk. If an individual prefers present to future because of the uncertainty of future, he hasn't ignore present for future for some positive interest rate.

C. From the Mises' viewpoint consuming means that we have time preference. If we indifferent between present and future we never consume and we delay consumption. But we say that consumption is not because of present preference. It is because of future present. If we want to reach to future (even if future is uncertain), it is necessary to consume now. We consume for future. If we don't consume now, we don't reach to future. Consuming now is better than consuming in future, because future is better than now. Future is more productive than now. Human wishes to have an everlasting life. He is wishful to future. He is waiting to future every moment (Bakhshi, 2007, P.70).

D. We agree that, a capital good now is preferred to the same capital good in future, because in future, that capital good hasn't any value. In future there will be another more productive capital good. On the base of this reasoning, no interest rate will be enough to delay using that same capital good to future. Because it is meaningless that we lend a laptop now for three or more the same laptop in 8 next years. Because in 8 next years there will be some better laptops. Therefore, we prefer future to present because there is progress. If there isn't any progress and economy had remained in stationary period we maybe preferred present to future. If economists believe to present preferring they can not describe the process of progress and growth.

6. Time Preference and Non-Renewable Resources:

In this section, According to Weber (2005) we use experimental data of ore extraction, for describing the economic managing of a nonrenewable resource to study the effects of time preference.

With annual data, the time path of ore extraction is the mine's output in each year: $\{x_0, x_1, ...x_T\}$. The value of the mine is the sum of discounted cash flows earned during its life, from 0 to T.

$$V = \sum_{t=0}^{T} R^t . C(x_t) \qquad (9)$$

where R=$\frac{1}{1+\rho}$ is the discount factor and ρ is the rate of time preference (equated by interest rate). For a numerical solution it is necessary to have a specified annual cash flow function. We assume that the cash flow function is:

$$C(x) = x_t^{0.8}$$

So we have:

$$V = \sum^{T} R^t . x_t^{0.8}$$

We maximize this function subject to:

$$y_t - y_{t+1} = x_t$$

where y_t and y_{t+1} are ore reserves in two periods of time. A numerical solution to our problem also needs to two conditions. Suppose the initial ore reserve is 1800 tons and the mine manager wants to extract all mine reserve for 12 years. So, the initial condition is $y_0 = 2000$, and the final condition is $y_T = y_{12} \geq 0$.

Now we want to set up our dynamic optimization problem in spreadsheet. We assume that market interest rate is 0.08. The formula in cell B3 converts the interest rate into the discount factor. Column C implements the constraint of problem (that is time path of the ore reserve). At the beginning of each year, the ore reserve equals the reserve at the beginning of the preceding year minus the ore extracted during the preceding year, $y_t - y_{t+1} = x_t$. The initial ore reserve, which is shown in cell C6, is assumed to be 1800 units. Solver uses an algorithm that needs to initial guess of the optimal time path of ore extraction. We first assume that ore is extracted evenly in each year, hence the value of 150 tones is included in cells B6 to B15. In the last column, DCF(t) stands for the discounted cash flow earned from ore extraction in year t, $R^t.x_t^{0.8}$. Finally, cell D18 contains the value of the mine, V, which is the sum of discounted cash flows earned during its operation.

The value of mine, V, depends on the time path of ore extraction. Worksheet 1 shows that the value of mine is 448.17 if 150 tones are extracted in each year. But, an even time path is not optimal when future cash flows are discounted. Applying Excel Solver yields the optimal time path of ore extraction that maximizes the value of the mine. We operate with Excel Solver for maximizing our present value. When we have completed the dialog box of Solver, and clicking on the Solver button in the top right corner we reach to:

Worksheet 2 shows the optimal time path of ore extraction. It is optimal to extract 580.69 tones in the first year, 395.2 tones in the second year and so on. The value of mine will be maximized at 504.06. As worksheet shows, the optimal time path of ore extraction is downward sloping because the discount factor provides an incentive to extract quickly.

A sensitivity analysis shows the influence of the time preference rate on the optimal solution. We want to check out the effect of a 0.2 increase in time preference rate on the optimal path of extraction.

Worksheet 3 shows the optimal time path of ore extraction with the higher time preference rate. The rise in the time preference rate increases the incentive to mine the ore quickly. Ore extraction increases to 684.72 tones in the first year and 425.01 tones in the second year and so on. It is shown that the optimal time path of extraction is steeper because future cash flows are discounted more.

Therefore the discount factor makes that worthwhile to extract quickly.

7. Conclusion:

As mentioned in previous sections, Time preference is an important variable in economics. In viewpoints of its founders there are alternative reasons that everyone want to prefer present over future. But we emphasized that time preference comes from irrationality. Pigou proposed the very fact that people possess defective "telescopic faculties". This line of reasoning was taken by Ramsey when he wanted to determine the highest possible rate of saving. Ramsey found out that the highest rate of saving will be occurred when time preference is vanished. If we believe to present preference, we couldn't describe development process in human life.

As we saw, discounting (via interest rate) has a negative effect on nonrenewable resources. With a high interest rate, the extraction process will be deeper and thus the ore will be finish sooner. It also affects the renewable resources.

The practical implications are self-evident: if time preference does lead people to save too little, perhaps the government should step in and force them to save. As Pigou concluded, "there is wide agreement that the State should protect the interests of the future in some degree against the effects of our irrational discounting and of our preferences for ourselves over our descendants." (Pigou, 1920: p.29).

Thus it is necessary to make some suitable policies for reducing social time preference.

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Worksheet 1:Values of mine based on the time path of ore extraction

	A	B	C	D
1	α=	0.8		
2	ρ=	0.08		
3	R=	0.9259259		
4				
5	Time	x(t)	y(t)	DCF(t)
6	0	150	1800	55.06
7	1	150	1650	50.99
8	2	150	1500	47.21
9	3	150	1350	43.71
10	4	150	1200	40.47
11	5	150	1050	37.48
12	6	150	900	34.70
13	7	150	750	32.13
14	8	150	600	29.75
15	9	150	450	27.55
16	10	150	300	25.51
17	11	150	150	23.62
18	12		0	
19				
20			Present Value=	448.17
21				

Source: Authors

Worksheet 2: The optinal time path of ore extraction

	A	B	C	D
1	α=	0.80		
2	ρ=	0.08		
3	R=	0.93		
4				
5	Time	x(t)	y(t)	DCF(t)
6	0.00	580.98	1800.00	162.68
7	1.00	395.35	1219.02	110.70
8	2.00	268.23	823.66	75.16
9	3.00	183.32	555.43	51.32
10	4.00	124.21	372.11	34.80
11	5.00	85.12	247.90	23.82
12	6.00	57.72	162.79	16.16
13	7.00	39.30	105.07	11.00
14	8.00	26.74	65.76	7.49
15	9.00	18.20	39.02	5.10
16	10.00	12.39	20.82	3.47
17	11.00	8.43	8.43	2.36
18	12.00		0.00	
19				
20			Present Value=	504.06
21				

Source: Authors

Worksheet 3: The optimal time path of ore extraction with the higher time preference rate

	A	B	C	D
1	α=	0.80		
2	ρ=	0.10		
3	R=	0.91		
4				
5	Time	x(t)	y(t)	DCF(t)
6	0.00	684.75	1800.00	185.54
7	1.00	425.00	1115.25	115.16
8	2.00	263.98	690.25	71.53
9	3.00	163.60	426.27	44.35
10	4.00	101.84	262.66	27.59
11	5.00	63.22	160.82	17.13
12	6.00	39.25	97.60	10.63
13	7.00	24.37	58.35	6.60
14	8.00	15.13	33.98	4.10
15	9.00	9.39	18.85	2.55
16	10.00	5.83	9.46	1.58
17	11.00	3.62	3.62	0.98
18	12.00		0.00	
19				
20			Present Value=	487.74
21				

Source: Authors